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Global Stability for the Leslie-Gower Predator-Prey System with Time-Delay

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Abstract

In this paper, we are concerned with the dynamical behavior of the Leslie-Gower predator-prey system with time delay. First of all, we discuss the global stability for the Leslie-Gower predator-prey system without time delay. Next, we study the change of the global stability for the Leslie-Gower predator-prey system with time delay. Finally, we illustrate our results by some examples.

Keywords: Leslie-Gower predator-prey system, global stability, delay, Lyapunov functional.

1 Introduction

One of the most interesting topics in a Predator-Prey system is the global stability of the Predator-Prey system. The global stability analysis for the Predator-Prey system without delay has been done by many authors. Most of them use the following methods to prove global stability of a Predator-Prey system without delay. The first method is to construct a Lyapunov function [1, 2, 6, 7, 8, 10, 13, 14, 21, 23, 28]. The second method is to employ the Dulac Criterion to eliminate the existence of periodic orbits and then use the Poincaré-Bendixson Theorem to analyze the global stability of the unique positive equilibrium [8, 10, 12, 13, 14, 15, 17]. The third method is the comparison method. In [4, 10, 11, 17, 22], the authors obtain an auxiliary system by "mirror" reflection, analyze the global stability of the auxiliary system, then compare the trajectories of the system with those of the auxiliary system. The fourth method is the limit cycle stability analysis [3, 4, 10, 11, 12, 17]. The method is to prove there is no periodic orbit in the system by contradiction. Suppose there exists a periodic orbit, and prove that all periodic orbits are orbitally asymptotically stable. Then we are able to interpret the uniqueness of the limit cycle. If the positive equilibrium is locally asymptotically stable, then we obtain the contradiction. That is, there is no periodic orbit for the system and the positive equilibrium is globally asymptotically stable.

In recent years, many authors extended their research to discussing a delayed Predator-Prey system. In [5, 9, 19, 20, 26, 30, 31], the global stability of the system with time-delay is analyzed by constructing a Lyapunov functional. Sanyi

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[27] employ the theory of competitive systems, compound matrices, and stability of periodic orbits. Yasuhisa [25] used the extended LaSalle's invariance principle.

In this paper, we were concerned about the Leslie-Gower Predator-Prey system. For this system without delay, references [16] and [11] analyzed the global stability by constructing a Lyapunov functional or Comparison method, respectively. We extend this to analyze the global stability of the Leslie-Gower Predator-Prey system with a single delay by constructing a Lyapunov functional, and we illustrate our results by some examples.

2 The model without delay

Consider the Leslie-Gower predator-prey system without time delay modeled by

$$\begin{aligned} \dot{x_1}(t) &= x_1(t)[r_1 - b_1x_1(t) - a_1x_2(t)] \\ \dot{x_2}(t) &= x_2(t)\left[r_2 - a_2\frac{x_2(t)}{x_1(t)}\right] \end{aligned} \tag{2.1}$$

with the initial condition

$$x_1(0) > 0, \quad x_2(0) > 0 \tag{2.2}$$

where r_1, r_2, a_1, a_2, and b_1 are positive constants, and x_1 and x_2 denote the densities of prey and predator population, respectively.

Clearly, $\widehat{E} \equiv (r_1/b_1, 0)$ is an equilibrium point and $E^* \equiv (x_1^*, x_2^*)$ is the unique positive equilibrium point in the first quadrant for the system (2.1) with the initial condition (2.2), where

$$x_1^* = \frac{r_1a_2}{a_1r_2 + a_2b_1}, \quad x_2^* = \frac{r_1r_2}{a_1r_2 + a_2b_1} \tag{2.3}$$

It follows from (2.3) that

$$r_2x_1^* = a_2x_2^*, \quad a_1x_2^* + b_1x_1^* = r_1 \tag{2.4}$$

Firstly, we discuss the local behavior of equilibrium points of the system (2.1) with the initial condition (2.2) by the Hartman-Grobman Theorem. The Jacobian matrix of the system (2.1) takes the form

$$J = \begin{bmatrix} r_1 - 2b_1x_1(t) - a_1x_2(t) & -a_1x_1(t) \\ a_2\dfrac{x_2^2(t)}{x_1^2(t)} & r_2 - \dfrac{2a_2x_2(t)}{x_1(t)} \end{bmatrix}$$

The Jacobian matrix of the system (2.1) at $\widehat{E}$ is

$$\widehat{J} = \begin{bmatrix} -r_1 & -\dfrac{a_1r_1}{b_1} \\ 0 & r_2 \end{bmatrix}$$

Since $\det(\widehat{J}) = -r_1 r_2 < 0$, the equilibrium point $\widehat{E}$ of (2.1) is a saddle point and the stable manifold is

$$\Gamma_1 = \{(x_1, x_2) | x_1 > 0,\ x_2 = 0\}$$

On the other hand, the Jacobian matrix of the system (2.1) at E^* is

$$J^* = \begin{bmatrix} -b_1 x_1^* & -a_1 x_1^* \\ a_2 \dfrac{(x_2^*)^2}{(x_1^*)^2} & -\dfrac{a_2 x_2^*}{x_1^*} \end{bmatrix}$$

Therefore,

$$\begin{aligned} \det(J^*) &= b_1 a_2 x_2^* + a_1 a_2 \frac{(x_2^*)^2}{x_1^*} \\ \mathrm{trace}(J^*) &= -b_1 x_1^* - \frac{a_2 x_2^*}{x_1^*} \end{aligned}$$

Since $\det(J^*) > 0$ and $\mathrm{trace}(J^*) < 0$, the equilibrium point E^* of (2.1) is locally asymptotically stable.

Lemma 2.1 *All solutions* $(x_1(t), x_2(t))$ *of the system* (2.1) *with the initial condition* (2.2) *are positive and bounded.*

Proof. Firstly, we want to show that all solutions $(x_1(t), x_2(t))$ of the system (2.1) with the initial condition (2.2) are positive. That is, if $(x_1(0), x_2(0))$ is in the first quadrant, then $(x_1(t), x_2(t))$ is also in the first quadrant for all $t \geq 0$. Divide the first quadrant into four regions I-IV which are defined as:

$$\begin{aligned} \mathrm{I} &= \{(x_1, x_2) \,|\, r_1 - b_1 x_1 - a_1 x_2 > 0, r_2 x_1 - a_2 x_2 > 0, x_1 > 0, x_2 > 0\} \\ \mathrm{II} &= \{(x_1, x_2) \,|\, r_1 - b_1 x_1 - a_1 x_2 < 0, r_2 x_1 - a_2 x_2 > 0, x_1 > 0, x_2 > 0\} \\ \mathrm{III} &= \{(x_1, x_2) \,|\, r_1 - b_1 x_1 - a_1 x_2 < 0, r_2 x_1 - a_2 x_2 < 0, x_1 > 0, x_2 > 0\} \\ \mathrm{IV} &= \{(x_1, x_2) \,|\, r_1 - b_1 x_1 - a_1 x_2 > 0, r_2 x_1 - a_2 x_2 < 0, x_1 > 0, x_2 > 0\} \end{aligned}$$

See Figure 2.1. Consider the following two cases:

(a) $(x_1(0), x_2(0))$ is near the positive x_1-axis;

(b) $(x_1(0), x_2(0))$ is near the positive x_2-axis;

In case (a), the initial point $(x_1(0), x_2(0))$ is in region I or II. Since $\dot{x}_2$ is positive in region I or II, the solution $(x_1(t), x_2(t))$ with the initial point $(x_1(0), x_2(0))$ will run away along the positive x_1-axis. In case (b), the initial point $(x_1(0), x_2(0))$ is

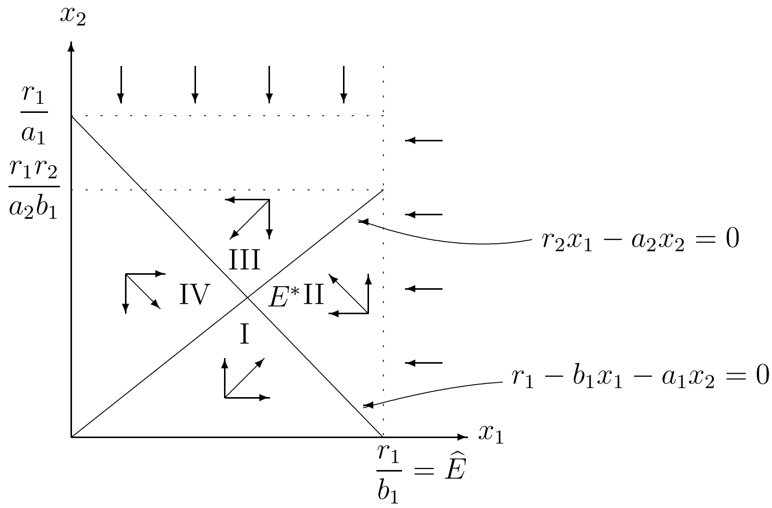

Figure 2.1. Schematic diagram for the proof of Lemma 2.1, where $L = \dfrac{r_1}{a_1}$.

in region III or IV. Since $\dot{x}_1$ is positive in region IV, the solution $(x_1(t), x_2(t))$ with the initial point $(x_1(0), x_2(0))$ will run away along the positive x_2-axis. Now, we want to show that if the initial point $(x_1(0), x_2(0))$ starts in III, then the trajectory of the solution $(x_1(t), x_2(t))$ will go into region IV. That is, the trajectory of the solution $(x_1(t), x_2(t))$ will not stay in region III nor go to the x_2-axis. Suppose that the trajectory finally stays at some point $(\overline{x}_1, \overline{x}_2)$ in region III, then $(\overline{x}_1, \overline{x}_2)$ will be an equilibrium point of the system (2.1). This is contradictory. Therefore any solution $(x_1(t), x_2(t))$ which starts in region III will not stay in it. On the other hand, if the trajectory in region III approaches the x_2-axis, then $\dot{x}_1 \to 0$ and $\dot{x}_2 \to -\infty$ as $x_1 \to 0$. Hence there is a $t_1 > 0$ such that $(x_1(t), x_2(t))$ is in region IV whenever $t \geq t_1$. Therefore, by the above discussion, we know that all solutions $(x_1(t), x_2(t))$ are positive.

Secondly, we want to show that all solutions $(x_1(t), x_2(t))$ of the system (2.1) with the initial condition (2.2) are bounded. We know $\dot{x}_1 < 0$ for $x_1 \geq r_1/b_1$ and $x_2 > 0$. Hence for solutions $(x_1(t), x_2(t))$ of the system (2.1) with the initial point $(x_1(0), x_2(0))$ and $x_1(0) \geq r_1/b_1$, there exists a $T_1 > 0$ such that $x_1(t) < r_1/b_1$ for $t > T_1$. Suppose that $x_2 \geq L \equiv \max\{r_1/a_1,\ r_1 r_2/a_2 b_1\}$ and $x_1 < r_1/b_1$. Now we want to show that there exists a $T_2 > 0$ such that $x_2(t) < L$ for $t > T_2$ whenever $x_1(0) < r_1/b_1$ and $x_2(0) \geq L$. If $L = r_1/a_1$, then $x_2 \geq r_1/a_1 > r_1 r_2/a_2 b_1$ and

$$\begin{aligned}
\dot{x}_2 &= x_2 \left[r_2 - \frac{a_2 x_2}{x_1} \right] \\
&\leq x_2 \left[r_2 - \frac{r_1 r_2}{b_1 x_1} \right]
\end{aligned}$$

$$= \quad x_2 \left[\frac{r_2(b_1 x_1 - r_1)}{b_1 x_1} \right] < 0$$

See Figure 2.1. On the other hand, if $L = r_1 r_2 / a_2 b_1$, then $x_2 \geq r_1 r_2 / a_2 b_1$, and $\dot{x}_2 < 0$. See Figure 2.2. Hence, by the above discussion, we know that for solutions $(x_1(t), x_2(t))$ of the system (2.1) with the initial point $(x_1(0), x_2(0))$ and $x_1(0) < r_1/b_1$, $x_2(0) \geq L$, there exists a $T_2 > 0$ such that $x_2(t) < L$ for $t > T_2$. So $x_1(t) < r_1/b_1$ and $x_2(t) < L$ for $t > T \equiv \max\{T_1, T_2\}$. That is, all solutions $(x_1(t), x_2(t))$ are bounded. ■

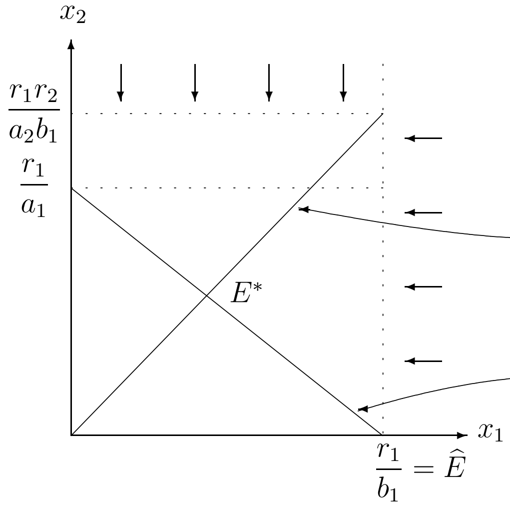

Figure 2.2. Schematic diagram for the proof of Lemma 2.1, where $L = \dfrac{r_1 r_2}{a_2 b_1}$.

Theorem 2.2 *The unique positive equilibrium point E^* of the system* (2.1) *is globally asymptotically stable.*

Proof. Consider

$$H(x_1, x_2) = \frac{1}{x_1 x_2}, \qquad x_1 > 0, \quad x_2 > 0$$

Then

$$\begin{aligned}
\nabla \cdot (Hf) &= \frac{\partial}{\partial x_1} \{ H \cdot [x_1 (r_1 - b_1 x_1 - a_1 x_2)] \} + \frac{\partial}{\partial x_2} \left\{ H \cdot \left[x_2 \left(r_2 - a_2 \frac{x_2}{x_1} \right) \right] \right\} \\
&= -\frac{x_1}{x_1^2 x_2}(r_1 - b_1 x_1 - a_1 x_2) + \frac{1}{x_1 x_2}(r_1 - 2 b_1 x_1 - a_1 x_2) \\
&\quad -\frac{x_2}{x_1 x_2^2} \left(r_2 - a_2 \frac{x_2}{x_1} \right) + \frac{1}{x_1 x_2} \left(r_2 - \frac{2 a_2 x_2}{x_1} \right) \\
&= -\frac{b_1}{x_2} - \frac{a_2}{x_1^2} < 0
\end{aligned}$$

Hence by Dulac's Criterion, there is no closed orbit in the first quadrant. From above, we see that E^* is locally asymptotically stable. By Lemma 2.1 and the

Poincaré-Bendixson theorem, it suffices to show that the unique positive equilibrium point E^* is globally asymptotically stable in the first quadrant. ■

Remark 2.3 *a) In [16] the same result with Theorem* 2.2 *was obtained via the Lyapunov functional*

$$V(x_1, x_2) = \ln \frac{x_1}{x_1^*} + \frac{x_1^*}{x_1} + \frac{a_1 x_1^*}{a_2} \left(\ln \frac{x_2}{x_2^*} + \frac{x_2^*}{x_2} \right)$$

b) We also can use the method: "Stable limit cycle analysis" to prove this theorem.

3 The model with delay

Consider the Leslie-Gower predator-prey system with time delay τ modelled by

$$\begin{aligned} \dot{x_1}(t) &= x_1(t)[r_1 - b_1 x_1(t-\tau) - a_1 x_2(t)] \\ \dot{x_2}(t) &= x_2(t) \left[r_2 - a_2 \frac{x_2(t)}{x_1(t)} \right] \end{aligned} \tag{3.1}$$

with the initial conditions

$$\begin{aligned} x_1(\theta) &= \phi(\theta) \geq 0, \theta \in [-\tau, 0], \phi \in C([-\tau, 0], R) \\ x_1(0) &> 0, \ \ x_2(0) \ > \ 0 \end{aligned} \tag{3.2}$$

where $r_1, r_2, a_1, a_2, b_1,$ and τ are positive constants, and x_1 and x_2 denote the densities of prey and predator population, respectively.

Lemma 3.1 *Every solution of the system* (3.1) *with the initial conditions* (3.2) *exists in the interval* $[0, \infty)$ *and remains positive for all* $t \geq 0$.

Proof. It is true because

$$\begin{aligned} x_1(t) &= x_1(0) \exp \left\{ \int_0^t [r_1 - b_1 x_1(s-\tau) - a_1 x_2(s)] \, ds \right\} \\ x_2(t) &= x_2(0) \exp \left\{ \int_0^t \left[r_2 - a_2 \frac{x_2(s)}{x_1(s)} \right] \, ds \right\} \end{aligned}$$

and $x_i(0) > 0$ for $i = 1, 2$. ■

Lemma 3.2 *Let* $(x_1(t), x_2(t))$ *denote the solution of* (3.1) *with the initial condition* (3.2). *Then*

$$0 < x_i(t) \leq M_i, \quad i = 1, 2 \tag{3.3}$$

eventually for all large t, where

$$M_1 = \frac{r_1}{b_1} e^{r_1 \tau} \tag{3.4}$$

$$M_2 = \frac{r_2}{a_2} M_1 \tag{3.5}$$

Proof. We want to show that there exists a $T > 0$ such that $x_1(t) \leq M_1$ for $t > T$. By Lemma 3.1, we know that solutions of the system (3.1) with the initial condition (3.2) are positive, and hence, by (3.1),

$$\begin{aligned} \dot{x_1}(t) &= x_1(t)[r_1 - b_1 x_1(t-\tau) - a_1 x_2(t)] \\ &\leq x_1(t)[r_1 - b_1 x_1(t-\tau)] \end{aligned} \tag{3.6}$$

Taking $M_1^* = r_1(1+k_1)/b_1$, $0 < k_1 < e^{r_1 \tau} - 1$. Suppose $x_1(t)$ is not oscillatory about M_1^*. That is, there exists a $T_0 > 0$ such that either

$$x_1(t) > M_1^* \quad \text{for} \quad t > T_0 \tag{3.7}$$

or

$$x_1(t) \leq M_1^* \quad \text{for} \quad t > T_0 \tag{3.8}$$

If (3.8) holds, then for $t > T_0$

$$x_1(t) \leq M_1^* = \frac{r_1(1+k_1)}{b_1} < \frac{r_1}{b_1} e^{r_1 \tau} = M_1$$

That is, (3.3) holds. Suppose (3.7) holds. Equation (3.6) implies that for $t > T_0 + \tau$

$$\begin{aligned} \dot{x}_1(t) &\leq x_1(t)[r_1 - b_1 x_1(t-\tau)] \\ &< -k_1 r_1 x_1(t) \end{aligned}$$

It follows that

$$\int_{T_0+\tau}^{t} \frac{\dot{x}_1(s)}{x_1(s)}\, ds < \int_{T_0+\tau}^{t} -k_1 r_1 \, ds = -k_1 r_1 (t - T_0 - \tau)$$

Then $0 < x_1(t) < x_1(T_0+\tau)\, e^{-k_1 r_1 (t - T_0 - \tau)} \to 0$ as $t \to \infty$. That is, $\lim_{t\to\infty} x_1(t) = 0$ by the Squeeze Theorem. This contradicts (3.7). Therefore, there must exist a $T_1 > T_0$ such that $x_1(T_1) \leq M_1^*$. If $x_1(t) \leq M_1^*$ for all $t \geq T_1$, then (3.3) follows. If not, then there must exist a $T_2 > T_1$ such that T_2 is the first time at which $x_1(T_2) > M_1^*$. Therefore, there exists a $T_3 > T_2$ such that T_3 is the first time at which $x_1(T_3) < M_1^*$ by the above discussion. By the above, we know that $x_1(T_1) \leq M_1^*$, $x_1(T_2) > M_1^*$, and $x_1(T_3) \leq M_1^*$ where $T_1 < T_2 < T_3$. Then, by the Intermediate Value Theorem, there exist T_4 and T_5 such that

$$x_1(T_4) = M_1^*, \quad T_1 \leq T_4 < T_2$$

$$x_1(T_5) = M_1^*, \quad T_2 \le T_5 < T_3$$

and $x_1(t) > M_1^*$ for $T_4 < t < T_5$. Hence there is a $T_6 \in (T_4, T_5)$ such that $x_1(T_6)$ is an arbitrary local maximum, and hence it follows from (3.6) that

$$0 = \dot{x}_1(T_6) \le x_1(T_6)[r_1 - b_1 x_1(T_6 - \tau)]$$

and this implies

$$x_1(T_6 - \tau) \le \frac{r_1}{b_1}$$

Integrating both sides of (3.6) on the interval $[T_6 - \tau, T_6]$, we have

$$\ln\left[\frac{x_1(T_6)}{x_1(T_6 - \tau)}\right] = \int_{T_6-\tau}^{T_6} \frac{\dot{x}_1(s)}{x_1(s)}\, ds \le \int_{T_6-\tau}^{T_6} [r_1 - b_1 x_1(s - \tau)]\, ds \le r_1\tau$$

It follows that

$$x_1(T_6) \le x_1(T_6 - \tau)\, e^{r_1\tau} \le \frac{r_1}{b_1} e^{r_1\tau} = M_1$$

Since $x_1(T_6)$ is local maximum of $x_1(t)$ and $x_1(T_6) \le M_1$, $x_1(t) \le M_1$ where t is near T_6. Since $x_1(T_6)$ is an arbitrary local maximum of $x_1(t)$, we can conclude that there exists a $T > 0$ such that

$$x_1(t) \le M_1 \quad \text{for} \quad t \ge T \tag{3.9}$$

Suppose $x_1(t)$ is oscillatory about M_1^*; for this case, the proof is similar to the above one. Now, we want to show that $x_2(t)$ is bounded above by M_2 eventually for all large t. By (3.9), it follows that for $t > T$

$$\begin{aligned}
\dot{x}_2(t) &= x_2(t)[r_2 - a_2 \frac{x_2(t)}{x_1(t)}] \\
&\le x_2(t)[r_2 - \frac{a_2}{M_1} x_2(t)] \\
&= r_2 x_2(t)[1 - \frac{a_2}{r_2 M_1} x_2(t)] \\
&= r_2 x_2(t)[1 - \frac{x_2(t)}{\frac{r_2 M_1}{a_2}}]
\end{aligned}$$

Therefore, $x_2(t) \le r_2 M_1 / a_2 = M_2$ for $t > T$. This completes the proof. ■

Lemma 3.3 *Suppose that the system* (3.1) *satisfies*

$$r_1 - a_1 M_2 > 0 \tag{3.10}$$

where M_2 *is defined by* (3.5). *Then the system* (3.1) *is uniformly persistent. That is, there exist* m_1, m_2, *and* $T^* > 0$ *such that* $m_i \le x_i(t) \le M_i$ *for* $t \ge T^*$, $i = 1, 2$.

Proof. By Lemma 3.2, equation (3.1) follows that for $t \geq T + \tau$

$$\dot{x_1}(t) \;\; \geq \;\; x_1(t)[r_1 - b_1M_1 - a_1M_2] \tag{3.11}$$

Integrating both sides of (3.11) on $[t - \tau, t]$, where $t \geq T + \tau$, then we have

$$x_1(t) \;\; \geq \;\; x_1(t-\tau)\; e^{(r_1 - b_1M_1 - a_1M_2)\tau}$$

That is,

$$x_1(t-\tau) \;\; \leq \;\; x_1(t)\; e^{-(r_1 - b_1M_1 - a_1M_2)\tau} \tag{3.12}$$

It follows from (3.1) that for $t \geq T + \tau$

$$\begin{aligned} \dot{x_1}(t) &= x_1(t)[r_1 - b_1x_1(t-\tau) - a_1x_2(t)] \\ &\geq x_1(t)[r_1 - a_1M_2 - b_1e^{-(r_1 - b_1M_1 - a_1M_2)\tau}\; x_1(t)] \\ &= (r_1 - a_1M_2)\; x_1(t)\; [1 - \frac{b_1e^{-(r_1 - b_1M_1 - a_1M_2)\tau}}{r_1 - a_1M_2}\; x_1(t)] \\ &= (r_1 - a_1M_2)\; x_1(t)\; [1 - \frac{x_1(t)}{\frac{r_1 - a_1M_2}{b_1}\; e^{(r_1 - b_1M_1 - a_1M_2)\tau}}] \end{aligned}$$

It follows that

$$\liminf_{t\to\infty} x_1(t) \geq \frac{r_1 - a_1M_2}{b_1}\; e^{(r_1 - b_1M_1 - a_1M_2)\tau} \equiv \overline{m}_1$$

and $\overline{m_1} > 0$ by (3.10). So, for large t, $x_1(t) > \overline{m}_1/2 \equiv m_1 > 0$. It follows that

$$\begin{aligned} \dot{x_2}(t) &\geq x_2(t)[r_2 - \frac{a_2}{m_1}x_2(t)] \\ &= r_2x_2(t)[1 - \frac{a_2}{r_2m_1}x_2(t)] \\ &= r_2x_2(t)[1 - \frac{x_2(t)}{\frac{r_2m_1}{a_2}}] \end{aligned}$$

Then

$$\liminf_{t\to\infty} x_2(t) \geq \frac{r_2m_1}{a_2} \equiv \overline{m}_2$$

So, for large t, $x_2(t) > \overline{m}_2/2 \equiv m_2 > 0$. Let

$$D = \{(x_1, x_2) \,|\, m_1 \leq x_1 \leq M_1,\; m_2 \leq x_2 \leq M_2\}$$

Then D is a bounded compact region in R^2_+ that has a positive distance from coordinate hyperplanes. Hence we obtain that there exists a $T^* > 0$ such that if $t \geq T^*$, then every positive solution of system (3.1) with the initial conditions (3.2) eventually enters and remains in the region D, that is, system (3.1) is uniformly persistent. ■

Theorem 3.4 *If the delay* τ *satisfy*

$$r_1 - a_1 M_2 > 0 \tag{3.13}$$

$$b_1 M_1^2 \tau < 2x_1^* \tag{3.14}$$

$$b_1 m_1 M_1 (r_1 + b_1 x_1^*)\tau < 2x_1^*(b_1 m_1 - a_1 M_2 - a_1 x_2^*) \tag{3.15}$$

where $m_1, M_1,$ *and* M_2 *are defined in Lemmas* 3.2 *and* 3.3*, then the unique positive equilibrium* E^* *of the system* (3.1) *is globally asymptotically stable.*

Proof. Define $y(t) = (y_1(t), y_2(t))$ by

$$y_1(t) = \frac{x_1(t) - x_1^*}{x_1^*},\ y_2(t) = \frac{x_2(t) - x_2^*}{x_2^*}$$

From (3.1),

$$\dot{y_1}(t) = [1 + y_1(t)][-b_1 x_1^* y_1(t-\tau) - a_1 x_2^* y_2(t)] \tag{3.16}$$

$$\dot{y_2}(t) = [1 + y_2(t)]\left[\frac{r_2 x_1^* y_1(t) - a_2 x_2^* y_2(t)}{x_1^*(1 + y_1(t))}\right] \tag{3.17}$$

Let

$$V_1(y(t)) = \frac{1}{a_1 x_1^* x_2^*}\{y_1(t) - \ln[1 + y_1(t)]\} + \frac{1}{r_2 x_1^*}\{y_2(t) - \ln[1 + y_2(t)]\} \tag{3.18}$$

then we have from (3.16) and (3.17) that

$$\begin{aligned}
\dot{V}_1(y(t)) &= \frac{1}{a_1 x_1^* x_2^*} \cdot \frac{y_1(t)\dot{y}_1(t)}{1 + y_1(t)} + \frac{1}{r_2 x_1^*} \cdot \frac{y_2(t)\dot{y}_2(t)}{1 + y_2(t)} \\
&= -\frac{b_1}{a_1 x_2^*} y_1(t) y_1(t-\tau) - \frac{1}{x_1^*} y_1(t) y_2(t) + \frac{y_1(t) y_2(t)}{x_1^*[1 + y_1(t)]} - \frac{y_2^2(t)}{x_1^*[1 + y_1(t)]} \\
&= -\frac{b_1}{a_1 x_2^*} y_1(t) y_1(t-\tau) - \frac{y_1^2(t) y_2(t)}{x_1^*[1 + y_1(t)]} - \frac{y_2^2(t)}{x_1^*[1 + y_1(t)]} \\
&\leq -\frac{b_1}{a_1 x_2^*} y_1(t) y_1(t-\tau) + \frac{|y_2(t)| y_1^2(t)}{x_1^*[1 + y_1(t)]} - \frac{y_2^2(t)}{x_1^*[1 + y_1(t)]} \qquad (3.19)
\end{aligned}$$

By Lemma 3.3, there exists a $T^* > 0$ such that $m_i \leq x_i^*[1 + y_i(t)] = x_i(t) \leq M_i$ for $t > T^*$, $i = 1, 2$. Then (3.19) implies that

$$\begin{aligned}
\dot{V}_1(y(t)) &\leq -\frac{b_1}{a_1 x_2^*} y_1(t) y_1(t-\tau) + \frac{1}{m_1}|y_2(t)| y_1^2(t) - \frac{1}{M_1} y_2^2(t) \\
&\leq -\frac{b_1}{a_1 x_2^*} y_1(t) y_1(t-\tau) + \frac{1}{m_1}\left(1 + \frac{M_2}{x_2^*}\right) y_1^2(t) - \frac{1}{M_1} y_2^2(t)
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{b_1}{a_1x_2^*}y_1(t)[y_1(t) - \int_{t-\tau}^{t}\dot{y}_1(s)\ ds] + \frac{1}{m_1}\left(1+\frac{M_2}{x_2^*}\right)y_1^2(t) - \frac{1}{M_1}y_2^2(t) \\
&= -\left(\frac{b_1}{a_1x_2^*} - \frac{M_2}{m_1x_2^*} - \frac{1}{m_1}\right)y_1^2(t) - \frac{1}{M_1}y_2^2(t) \\
&\quad + \frac{b_1}{a_1x_2^*}y_1(t)\int_{t-\tau}^{t}[1+y_1(s)][-b_1x_1^*y_1(s-\tau) - a_1x_2^*y_2(s)]\ ds \\
&= -\left(\frac{b_1}{a_1x_2^*} - \frac{M_2}{m_1x_2^*} - \frac{1}{m_1}\right)y_1^2(t) - \frac{1}{M_1}y_2^2(t) \\
&\quad + \frac{b_1}{a_1x_2^*}\int_{t-\tau}^{t}[1+y_1(s)][-b_1x_1^*y_1(t)y_1(s-\tau) - a_1x_2^*y_1(t)y_2(s)]\ ds \\
&\leq -\left(\frac{b_1}{a_1x_2^*} - \frac{M_2}{m_1x_2^*} - \frac{1}{m_1}\right)y_1^2(t) - \frac{1}{M_1}y_2^2(t) \\
&\quad + \frac{b_1}{a_1x_2^*}\int_{t-\tau}^{t}[1+y_1(s)][b_1x_1^*|y_1(t)y_1(s-\tau)| + a_1x_2^*|y_1(t)y_2(s)|]\ ds \qquad (3.20)
\end{aligned}
$$

$$(3.21)$$

Then for $t \geq T^* + \tau \equiv \widehat{T}$, we have from (3.21) that

$$
\begin{aligned}
\dot{V}_1(y(t)) &\leq -\left(\frac{b_1}{a_1x_2^*} - \frac{M_2}{m_1x_2^*} - \frac{1}{m_1}\right)y_1^2(t) - \frac{1}{M_1}y_2^2(t) \\
&\quad + \frac{b_1M_1}{a_1x_1^*x_2^*}\int_{t-\tau}^{t}[b_1x_1^*|y_1(t)||y_1(s-\tau)| + a_1x_2^*|y_1(t)||y_2(s)|]\ ds \\
&\leq -\left(\frac{b_1}{a_1x_2^*} - \frac{M_2}{m_1x_2^*} - \frac{1}{m_1}\right)y_1^2(t) - \frac{1}{M_1}y_2^2(t) + \frac{b_1M_1}{a_1x_1^*x_2^*}\left[\frac{b_1x_1^*\tau}{2}y_1^2(t)\right. \\
&\quad \left. + \frac{b_1x_1^*}{2}\int_{t-\tau}^{t}y_1^2(s-\tau)\ ds + \frac{a_1x_2^*\tau}{2}y_1^2(t) + \frac{a_1x_2^*}{2}\int_{t-\tau}^{t}y_2^2(s)\ ds\right] \\
&= -\left(\frac{b_1}{a_1x_2^*} - \frac{M_2}{m_1x_2^*} - \frac{b_1^2M_1\tau}{2a_1x_2^*} - \frac{b_1M_1\tau}{2x_1^*} - \frac{1}{m_1}\right)y_1^2(t) - \frac{1}{M_1}y_2^2(t) \\
&\quad + \frac{b_1^2M_1}{2a_1x_2^*}\int_{t-\tau}^{t}y_1^2(s-\tau)\ ds + \frac{b_1M_1}{2x_1^*}\int_{t-\tau}^{t}y_2^2(s)\ ds \qquad (3.22)
\end{aligned}
$$

Let

$$
\begin{aligned}
V_2(y(t)) &= \frac{b_1^2M_1}{2a_1x_2^*}\int_{t-\tau}^{t}\int_{s}^{t}y_1^2(\gamma-\tau)\ d\gamma\ ds \\
&\quad + \frac{b_1M_1}{2x_1^*}\int_{t-\tau}^{t}\int_{s}^{t}y_2^2(\gamma)\ d\gamma\ ds \qquad (3.23)
\end{aligned}
$$

then

$$\dot{V}_2(y(t)) = \frac{b_1^2 M_1 \tau}{2a_1 x_2^*} y_1^2(t-\tau) - \frac{b_1^2 M_1}{2a_1 x_2^*} \int_{t-\tau}^{t} y_1^2(s-\tau)\, ds$$
$$+ \frac{b_1 M_1 \tau}{2x_1^*} y_2^2(t) - \frac{b_1 M_1}{2x_1^*} \int_{t-\tau}^{t} y_2^2(s)\, ds \tag{3.24}$$

and then we have from (3.22) and (3.24) that for $t \geq \widehat{T}$

$$\dot{V}_1(y(t)) + \dot{V}_2(y(t)) \leq -\left(\frac{b_1}{a_1 x_2^*} - \frac{M_2}{m_1 x_2^*} - \frac{b_1^2 M_1 \tau}{2a_1 x_2^*} - \frac{b_1 M_1 \tau}{2x_1^*} - \frac{1}{m_1} \right) y_1^2(t)$$
$$- \left(\frac{1}{M_1} - \frac{b_1 M_1 \tau}{2x_1^*} \right) y_2^2(t) + \frac{b_1^2 M_1 \tau}{2a_1 x_2^*} y_1^2(t-\tau) \tag{3.25}$$

Let

$$V_3(y(t)) = \frac{b_1^2 M_1 \tau}{2a_1 x_2^*} \int_{t-\tau}^{t} y_1^2(s)\, ds \tag{3.26}$$

then

$$\dot{V}_3(y(t)) = \frac{b_1^2 M_1 \tau}{2a_1 x_2^*} y_1^2(t) - \frac{b_1^2 M_1 \tau}{2a_1 x_2^*} y_1^2(t-\tau) \tag{3.27}$$

Now define a Lyapunov functional $V(y(t))$ as

$$V(y(t)) = V_1(y(t)) + V_2(y(t)) + V_3(y(t)) \tag{3.28}$$

then we have from (3.25) and (3.27) that for $t \geq \widehat{T}$

$$\dot{V}(y(t)) \leq -\left(\frac{b_1}{a_1 x_2^*} - \frac{M_2}{m_1 x_2^*} - \frac{b_1^2 M_1 \tau}{a_1 x_2^*} - \frac{b_1 M_1 \tau}{2x_1^*} - \frac{1}{m_1} \right) y_1^2(t)$$
$$- \left(\frac{1}{M_1} - \frac{b_1 M_1 \tau}{2x_1^*} \right) y_2^2(t)$$
$$= -\frac{2x_1^*(b_1 m_1 - a_1 M_2 - a_1 x_2^*) - b_1 m_1 M_1 (r_1 + b_1 x_1^*)\tau}{2a_1 m_1 x_1^* x_2^*} y_1^2(t)$$
$$-\frac{2x_1^* - b_1 M_1^2 \tau}{2x_1^* M_1} y_2^2(t)$$
$$\equiv -\alpha\, y_1^2(t) - \beta\, y_2^2(t) \tag{3.29}$$

Then it follows from (3.14) and (3.15) that $\alpha > 0$ and $\beta > 0$. Let $w(s) = \widehat{N} s^2$ where $\widehat{N} = \min\{\alpha, \beta\}$; then w is nonnegative continuous on $[0, \infty]$, $w(0) = 0$, and $w(s) > 0$ for $s > 0$. It follows from (3.29) that for $t \geq \widehat{T}$

$$\dot{V}(y(t)) \leq -\widehat{N}\, [y_1^2(t) + y_2^2(t)] = -\widehat{N}\, |y(t)|^2 = -w(|y(t)|) \tag{3.30}$$

Now, we want to find a function u such that $V(y(t)) \geq u(|y(t)|)$. It follows from (3.18), (3.23), and (3.26) that

$$V(y(t)) \geq \frac{1}{a_1 x_1^* x_2^*}\{y_1(t) - \ln[1 + y_1(t)]\} + \frac{1}{r_2 x_1^*}\{y_2(t) - \ln[1 + y_2(t)]\} \qquad (3.31)$$

By the Taylor Theorem, we have that

$$y_i(t) - \ln[1 + y_i(t)] = \frac{y_i^2(t)}{2[1 + \theta_i(t)]^2} \qquad (3.32)$$

where $\theta_i(t) \in (0, y_i(t))$ or $(y_i(t), 0)$ for $i = 1, 2$.
Case 1 : If $0 < \theta_i(t) < y_i(t)$ for $i = 1, 2$, then

$$\frac{y_i^2(t)}{[1 + y_i(t)]^2} < \frac{y_i^2(t)}{[1 + \theta_i(t)]^2} < y_i^2(t) \qquad (3.33)$$

By Lemma 3.3, it follows that for $t \geq T^*$

$$m_i \leq x_i^*[1 + y_i(t)] = x_i(t) \leq M_i, \quad i = 1, 2 \qquad (3.34)$$

Then (3.33) implies that

$$\left(\frac{x_i^*}{M_i}\right)^2 y_i^2(t) \leq \frac{y_i^2(t)}{[1 + \theta_i(t)]^2} < y_i^2(t), \quad i = 1, 2 \qquad (3.35)$$

It follows from (3.31), (3.32), and (3.35) that for $t \geq T^*$

$$\begin{aligned} V(y(t)) &\geq \frac{1}{2a_1 x_1^* x_2^*}\frac{y_1^2(t)}{[1 + \theta_1(t)]^2} + \frac{1}{2r_2 x_1^*}\frac{y_2^2(t)}{[1 + \theta_2(t)]^2} \\ &\geq \frac{1}{2a_1 x_1^* x_2^*}\left(\frac{x_1^*}{M_1}\right)^2 y_1^2(t) + \frac{1}{2r_2 x_1^*}\left(\frac{x_2^*}{M_2}\right)^2 y_2^2(t) \\ &\geq \min\left\{\frac{1}{2a_1 x_1^* x_2^*}\left(\frac{x_1^*}{M_1}\right)^2, \frac{1}{2r_2 x_1^*}\left(\frac{x_2^*}{M_2}\right)^2\right\} [y_1^2(t) + y_2^2(t)] \\ &\equiv \widetilde{N}\ |y(t)|^2 \end{aligned}$$

Case 2 : If $-1 < y_i(t) < \theta_i(t) < 0$ for $i = 1, 2$, then

$$y_i^2(t) < \frac{y_i^2(t)}{[1 + \theta_i(t)]^2} < \frac{y_i^2(t)}{[1 + y_i(t)]^2} \qquad (3.36)$$

By (3.34), (3.36) implies that

$$y_i^2(t) < \frac{y_i^2(t)}{[1 + \theta_i(t)]^2} \leq \left(\frac{x_i^*}{m_i}\right)^2 y_i^2(t), \quad i = 1, 2 \qquad (3.37)$$

It follows from (3.31), (3.32), and (3.37) that for $t \geq T^*$

$$\begin{aligned} V(y(t)) &\geq \frac{1}{2a_1x_1^*x_2^*}\frac{y_1^2(t)}{[1+\theta_1(t)]^2} + \frac{1}{2r_2x_1^*}\frac{y_2^2(t)}{[1+\theta_2(t)]^2} \\ &> \frac{1}{2a_1x_1^*x_2^*}y_1^2(t) + \frac{1}{2r_2x_1^*}y_2^2(t) \\ &\geq \frac{1}{2a_1x_1^*x_2^*}\left(\frac{x_1^*}{M_1}\right)^2 y_1^2(t) + \frac{1}{2r_2x_1^*}\left(\frac{x_2^*}{M_2}\right)^2 y_2^2(t) \\ &\geq \widetilde{N}\ [y_1^2(t) + y_2^2(t)] \\ &= \widetilde{N}\ |y(t)|^2 \end{aligned}$$

Case 3 : If $0 < \theta_1(t) < y_1(t)$ and $-1 < y_2(t) < \theta_2(t) < 0$, then it follows from (3.31), (3.32), (3.35) and (3.37) that for $t \geq T^*$

$$\begin{aligned} V(y(t)) &\geq \frac{1}{2a_1x_1^*x_2^*}\frac{y_1^2(t)}{[1+\theta_1(t)]^2} + \frac{1}{2r_2x_1^*}\frac{y_2^2(t)}{[1+\theta_2(t)]^2} \\ &> \frac{1}{2a_1x_1^*x_2^*}\left(\frac{x_1^*}{M_1}\right)^2 y_1^2(t) + \frac{1}{2r_2x_1^*}y_2^2(t) \\ &\geq \frac{1}{2a_1x_1^*x_2^*}\left(\frac{x_1^*}{M_1}\right)^2 y_1^2(t) + \frac{1}{2r_2x_1^*}\left(\frac{x_2^*}{M_2}\right)^2 y_2^2(t) \\ &\geq \widetilde{N}\ [y_1^2(t) + y_2^2(t)] \\ &= \widetilde{N}\ |y(t)|^2 \end{aligned}$$

Case 4 : If $-1 < y_1(t) < \theta_1(t) < 0$ and $0 < \theta_2(t) < y_2(t)$, then it follows from (3.31), (3.32), (3.35) and (3.37) that for $t \geq T^*$

$$\begin{aligned} V(y(t)) &\geq \frac{1}{2a_1x_1^*x_2^*}\frac{y_1^2(t)}{[1+\theta_1(t)]^2} + \frac{1}{2r_2x_1^*}\frac{y_2^2(t)}{[1+\theta_2(t)]^2} \\ &> \frac{1}{2a_1x_1^*x_2^*}y_1^2(t) + \frac{1}{2r_2x_1^*}\left(\frac{x_2^*}{M_2}\right)^2 y_2^2(t) \\ &\geq \frac{1}{2a_1x_1^*x_2^*}\left(\frac{x_1^*}{M_1}\right)^2 y_1^2(t) + \frac{1}{2r_2x_1^*}\left(\frac{x_2^*}{M_2}\right)^2 y_2^2(t) \\ &\geq \widetilde{N}\ [y_1^2(t) + y_2^2(t)] \\ &= \widetilde{N}\ |y(t)|^2 \end{aligned}$$

Let $u(s) = \widetilde{N}s^2$, then u is nonnegative continuous on $[0, \infty)$, $u(0) = 0$, $u(s) > 0$ for

$s > 0$, and $\lim_{s\to\infty} u(s) = +\infty$. So, by case 1 $\sim$ case 4, we have

$$V(y(t)) \geq u(|y(t)|) \quad \text{for} \quad t \geq T^* \tag{3.38}$$

So the equilibrium point E^* of the system (3.1) is globally asymptotically stable.■

4 Example

We present below two simple examples to illustrate the procedures of applying our results.

Example 4.1 Consider the system

$$\begin{aligned} \dot{x_1}(t) &= x_1(t)[1 - 10x_1(t) - x_2(t)] \\ \dot{x_2}(t) &= x_2(t)\left[1 - \frac{2x_2(t)}{x_1(t)}\right] \end{aligned} \tag{4.1}$$

where $r_1 = r_2 = 1$, $a_1 = 1$, $a_2 = 2$, $b_1 = 10$, and $E^* = (2/21, 1/21)$. Then we conclude that the unique positive equilibrium point E^* of the system (4.1) is globally asymptotically stable by Theorem 2.2. The trajectory of the system (4.1) is depicted in Figure 4.1.

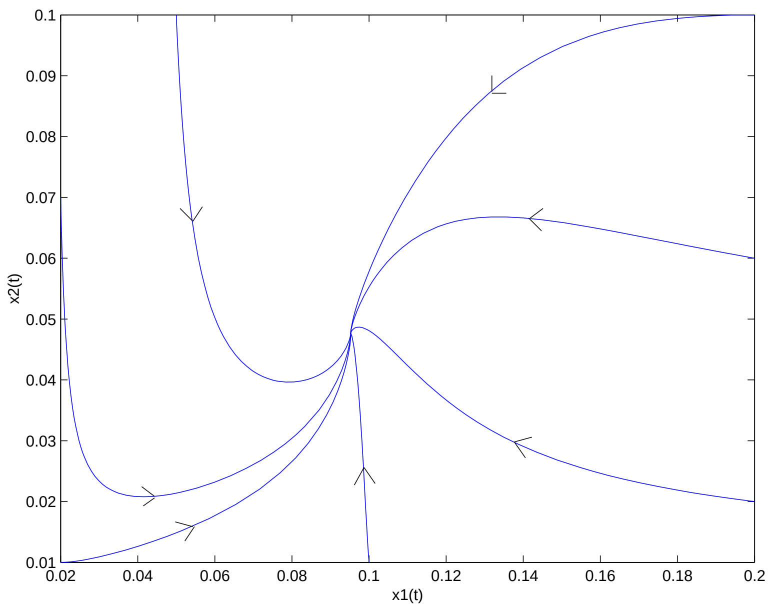

Figure 4.1. The trajectory of the system (4.1).

Example 4.2 Consider the system

$$\begin{aligned} \dot{x_1}(t) &= x_1(t)[1 - 10x_1(t-\tau) - x_2(t)] \\ \dot{x_2}(t) &= x_2(t)\left[1 - \frac{2x_2(t)}{x_1(t)}\right] \end{aligned} \tag{4.2}$$

where $r_1 = r_2 = 1$, $a_1 = 1$, $a_2 = 2$, $b_1 = 10$, and $E^* = (2/21, 1/21)$. Then

$$r_1 - a_1 M_2 = 0.9325 > 0$$

$$2x_1^* - b_1 M_1^2 \tau = 0.1358 > 0$$

$$2x_1^*(b_1 m_1 - a_1 M_2 - a_1 x_2^*) - b_1 m_1 M_1 (r_1 + b_1 x_1^*)\tau = 0.0239 > 0$$

whenever $\tau = 0.3$. Consequently, by Theorem 3.4, we conclude that the unique positive equilibrium point E^* of the system (4.2) is globally asymptotically stable. The trajectory of the system (4.2) is depicted in Figure 4.2.

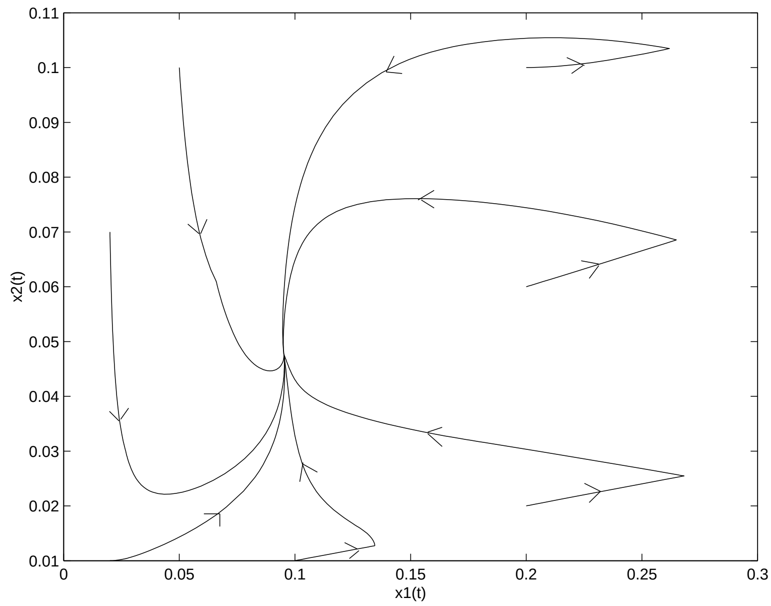

Figure 4.2. The trajectory of the system (4.2) with $\tau = 0.3$.

5 Conclusion

In this thesis, we obtain a sufficient condition for the global stability of the Leslie-Gower predator-prey system with time delay. We believe that the Leslie-Gower predator-prey system with time delay as follows will be an important topic for future study.

$$\begin{aligned} \dot{x_1}(t) &= x_1(t)[r_1 - b_1 x_1(t-\tau) - a_1 x_2(t)] \\ \dot{x_2}(t) &= x_2(t)\left[r_2 - a_2\frac{x_2(t)}{x_1(t-\tau)}\right] \end{aligned} \tag{5.1}$$

$$\begin{aligned} \dot{x_1}(t) &= x_1(t)[r_1 - b_1 x_1(t-\tau_1) - a_1 x_2(t-\tau_2)] \\ \dot{x_2}(t) &= x_2(t)\left[r_2 - a_2\frac{x_2(t-\tau_2)}{x_1(t-\tau_1)}\right] \end{aligned} \tag{5.2}$$

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時滯式 Leslie-Gower 捕食系統的整體穩定性

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摘要

本篇論文主要分析 Leslie-Gower 捕食系統的整體穩定性。首先，利用 Dulac's Criterion+Poincare'Bendixon Theorem 分析未具時滯參數之 Leslie-Gower 捕食系統的整體穩定性。緊接著，利用 Lyapunov Function 分析具時滯參數之 Leslie-Gower 捕食系統的整體穩定性。最後，用實例及電腦軌跡說明之。