Tunghai Science Vol. 10: 1−10
July, 2008

Nonequilibrium effective action for a charged particle coupled to quantized electromagnetic fluctuations in general covariant gauge

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Abstract

The nonequilibrium effective action for a charged particle interacting with the quantized electromagnetic fields is derived within the context of the closed-time-path formalism by integrating out field variables in general covariant gauge. The reduced density matrix of the charged particle is then obtained, thus enabling us to explore quantum decoherence phenomena of the charged particle in terms of the decoherence functional $\mathcal{W}$ by an interference experiment under the influence of electromagnetic quantum fluctuations. The issue of gauge invariance of the decoherence functional is discussed.
PACs numbers: 03.65.Yz, 05.40.a, 42.50.Lc

1 Introduction

The interaction of the charged particle and the electromagnetic fields has been studied quantum-mechanically in an open system approach [1]. If we focus on the dynamics of the charged particle, we may treat the charged particle as the system of interest, and the degrees of freedom of the electromagnetic fields as the environment. The influence of fields on the particle can be obtained by integrating out field variables within the context of the closed-time-path formalism [2, 3]. One way of observing these effects is via the interference experiment of the charged particle beam [4]. In the previous article [5], we employed the method of influence functional, and obtained the evolution of the reduced density matrix of the charged particle with self-consistent backreactions from the quantized electromagnetic fields. Under the classical approximation with prescribed trajectories of the charged particle, it was shown that the modulus of the exponent in the influence functional describes the change of the interference contrast in term of the decoherence functional, and its phase results in an overall shift of the interference pattern. However, the

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gauge in the previous study is the Coulomb gauge. In this paper, we would like to discuss the issue of gauge invariance of the decoherence functional. This gauge invariance can also be illustrated by explicitly computing the decoherence functional in general covariant gauge, and then comparing it with the result obtained in Coulomb gauge. Unless specified otherwise, the metric of the Minkowski space-time takes the convention of $\eta^{\mu\nu} = \text{diag}\,(1,-1,-1,-1)$, and the spatial components of a four-vector are denoted in bold-face.

2 Influence functional formalism

Gauge invariance for a charged particle that is minimally coupled to the electromagnetic fields requires the Lagrangian of the form [6]

$$L = \frac{m}{2}\dot{\mathbf{q}}^2 - V(\mathbf{q}) + \int d^3x \left[-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - j_\mu A^\mu\right], \tag{2.1}$$

where $\mathbf{q}$ describes the position of the charged particle, and j^μ is the current density,

$$j^\mu(x) = e\int d\tau\, u^\mu \delta^{(4)}(x - q(\tau)), \tag{2.2}$$

with $u^\mu = dq^\mu/d\tau$ being the four-velocity of the particle. In addition, $V(\mathbf{q})$ is an external potential. The field strength tensor $F^{\mu\nu}$ is defined by $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$. However, gauge invariance allows infinite copies of redundant degrees of freedom of vector potentials, related by the gauge transformation in the configuration space of the field. Some of this redundancy can be eliminated by introducing the gauge-fixing term [7],

$$\mathcal{L}_{\text{GF}} = -\frac{1}{2\xi}(\partial_\mu A^\mu)^2, \tag{2.3}$$

in general covariant gauge where ξ is an arbitrary real parameter. With the gauge-fixing term included, the Lagrangian then becomes

$$L[\mathbf{q}, A^\mu] = \frac{m}{2}\dot{\mathbf{q}}^2 - V(\mathbf{q}) + \int d^3x \left[-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2\xi}(\partial_\mu A^\mu)^2 - j_\mu A^\mu\right]. \tag{2.4}$$

Let ρ_e and ρ_A be the density matrices of the charged particle and the fields, respectively. We assume that the initial density matrix of the full system at time t_i can be factorized as

$$\rho(t_i) = \rho_e(t_i) \otimes \rho_A(t_i). \tag{2.5}$$

The fields are initially assumed in thermal equilibrium at temperature $T = 1/\beta$ with the density matrix $\rho_A(t_i)$ given by

$$\rho_A(t_i) = e^{-\beta H_A}, \tag{2.6}$$

where H_A is the Hamiltonian of the free electromagnetic fields. Then the zero-temperature limit, which corresponds to the initial vacuum state, can be reached by taking $T \to 0$. The particle-field system evolves unitarily according to

$$\rho(t_f) = U(t_f, t_i)\,\rho(t_i)\,U^{-1}(t_f, t_i) \tag{2.7}$$

with $U(t_f, t_i)$ the time evolution operator of the full system. Thereafter, the state at later times becomes entangled due to the interaction between them. We further assume that the interaction between the charged particle and fields is adiabatically switched on in the remote past with $t_i \to -\infty$, and then switched off in the remote future with $t_f \to \infty$. We then employ the closed-time-path formalism to describe the evolution of the density matrix of the charged particle and fields. In the context of the interference experiment, the effects of the quantized electromagnetic fields on the charged particle can be realized with the help of the diagonal elements of the reduced density matrix ρ_r after tracing out electromagnetic fields. The more detailed derivation can be found in Ref. [5]. Here we summarize the main results. In the coordinate basis the diagonal elements of the reduced density matrix ρ_r become

$$\rho_r(\mathbf{q}_f, \mathbf{q}_f, t_f) = \int d^3\mathbf{q}_1\, d^3\mathbf{q}_2\; \mathcal{J}(\mathbf{q}_f, \mathbf{q}_f, t_f; \mathbf{q}_1, \mathbf{q}_2, t_i)\,\rho_e(\mathbf{q}_1, \mathbf{q}_2, t_i)\,, \tag{2.8}$$

where the propagating function $\mathcal{J}(\mathbf{q}_f, \tilde{\mathbf{q}}_f, t_f; \mathbf{q}_1, \mathbf{q}_2, t_i)$ is defined as

$$\mathcal{J}(\mathbf{q}_f, \tilde{\mathbf{q}}_f, t_f; \mathbf{q}_1, \mathbf{q}_2, t_i) = \int_{\mathbf{q}_1}^{\mathbf{q}_f} \mathcal{D}\mathbf{q}^+ \int_{\mathbf{q}_2}^{\tilde{\mathbf{q}}_f} \mathcal{D}\mathbf{q}^- \exp\left[i\int_{t_i}^{t_f} dt\, \Big(L_e[\mathbf{q}^+] - L_e[\mathbf{q}^-]\Big)\right] \mathcal{F}[j^+_\mu, j^-_\nu]\,, \tag{2.9}$$

and the Lagrangian $L_e[\mathbf{q}]$ of the charged particle is given by

$$L_e[\mathbf{q}] = \frac{1}{2} m \dot{\mathbf{q}}^2 - V(\mathbf{q})\,. \tag{2.10}$$

Here we denote the quantities in the path integral running forward (backward) in time with the $+$ $(-)$ sign in the superscript. We introduce the influence functional $\mathcal{F}[j^+_\mu, j^-_\nu]$,

$$\begin{aligned}
\mathcal{F}[j^+_\mu, j^-_\nu] = \exp\bigg\{ -\frac{1}{2}\int d^4x \int d^4x' \Big[\; & j^+_\mu(x;\mathbf{q}^+(t))\,\big\langle A^{+\mu}(x) A^{+\nu}(x')\big\rangle\, j^+_\nu(x';\mathbf{q}^+(t')) \\
& - j^+_\mu(x;\mathbf{q}^+(t))\,\big\langle A^{+\mu}(x) A^{-\nu}(x')\big\rangle\, j^-_\nu(x';\mathbf{q}^-(t')) \\
& - j^-_\mu(x;\mathbf{q}^-(t))\,\big\langle A^{-\mu}(x) A^{+\nu}(x')\big\rangle\, j^+_\nu(x';\mathbf{q}^+(t')) \\
& + j^-_\mu(x;\mathbf{q}^-(t))\,\big\langle A^{-\mu}(x) A^{-\nu}(x')\big\rangle\, j^-_\nu(x';\mathbf{q}^-(t'))\; \Big] \bigg\}\,,
\end{aligned} \tag{2.11}$$

which contains full information about the influence of the quantized electromagnetic fields on the charged particle, and is a highly nonlocal object. The Green's functions of the vector potential are defined by

$$\begin{aligned}
\langle A^{+\mu}(x)A^{+\nu}(x')\rangle &= \langle A^{\mu}(x)A^{\nu}(x')\rangle\,\theta(t-t') + \langle A^{\nu}(x')A^{\nu}(x)\rangle\,\theta(t'-t)\,,\\
\langle A^{-\mu}(x)A^{-\nu}(x')\rangle &= \langle A^{\nu}(x')A^{\mu}(x)\rangle\,\theta(t-t') + \langle A^{\mu}(x)A^{\nu}(x')\rangle\,\theta(t'-t)\,,\\
\langle A^{+\mu}(x)A^{-\nu}(x')\rangle &= \langle A^{\nu}(x')A^{\mu}(x)\rangle \equiv \mathrm{Tr}\left\{\rho_{A_\mu}A^{\nu}(x')A^{\mu}(x)\right\}\,,\\
\langle A^{-\mu}(x)A^{+\nu}(x')\rangle &= \langle A^{\mu}(x)A^{\nu}(x')\rangle \equiv \mathrm{Tr}\left\{\rho_{A_\mu}A^{\mu}(x)A^{\nu}(x')\right\}\,,
\end{aligned} \tag{2.12}$$

and can be explicitly constructed as long as the electromagnetic fields are quantized in general covariant gauge. The retarded Green's function and Hadamard function of vector potentials are defined respectively by

$$G_R^{\mu\nu}(x-x') = i\,\theta(t-t')\,\langle\left[A^{\mu}(x),A^{\nu}(x')\right]\rangle\,, \tag{2.13}$$

$$G_H^{\mu\nu}(x-x') = \frac{1}{2}\,\langle\left\{A^{\mu}(x),A^{\nu}(x')\right\}\rangle\,. \tag{2.14}$$

Here the influence functional can be expressed in a more compact form in terms of its modulus and phase by

$$\mathcal{F}[j^+_\mu, j^-_\nu] = \exp\left\{\mathcal{W}[j^+_\mu, j^-_\nu] + i\,\Phi[j^+_\mu, j^-_\nu]\right\}, \tag{2.15}$$

with

$$\mathcal{W}[j^+_\mu, j^-_\nu] = -\frac{1}{2}\iint d^4x\,d^4x'\left[j^+_\mu(x;\mathbf{q}^+) - j^-_\mu(x;\mathbf{q}^-)\right]G_H^{\mu\nu}(x-x')\left[j^+_\nu(x';\mathbf{q}^+) - j^-_\nu(x';\mathbf{q}^-)\right], \tag{2.16}$$

$$\Phi[j^+_\mu, j^-_\nu] = \frac{1}{2}\iint d^4x\,d^4x'\left[j^+_\mu(x;\mathbf{q}^+) - j^-_\mu(x;\mathbf{q}^-)\right]G_R^{\mu\nu}(x-x')\left[j^+_\nu(x';\mathbf{q}^+) + j^-_\nu(x';\mathbf{q}^-)\right]. \tag{2.17}$$

Thus, the nonequilibrium effective action can be obtained as

$$S_{\mathrm{noneq}}[\mathbf{q}^+,\mathbf{q}^-] = \left\{\int_{t_i}^{t_f} dt\,\left(L_e[\mathbf{q}^+] - L_e[\mathbf{q}^-]\right)\right\} - i\ln\mathcal{F}[j^+_\mu, j^-_\nu]\,. \tag{2.18}$$

Although the nonequilibrium effective action of the charged particle is apparently gauge-dependent, any measurable effect obtained from this action must be invariant under the gauge transformation when the charged particle is in its on-shell condition.

Following Ref. [5], let us now consider the initial state vector $|\Psi(t_i)\rangle$ of the charged particle to be a coherent superposition of two localized states $|\psi_1\rangle$ and $|\psi_2\rangle$ along worldlines $\mathcal{C}_1$ and $\mathcal{C}_2$, respectively,

$$|\Psi(t_i)\rangle = |\psi_1(t_i)\rangle + |\psi_2(t_i)\rangle\,, \tag{2.19}$$

after they leave the beam splitter at the moment t_i. Its initial density matrix is then given by

$$\begin{aligned}
\rho_e(t_i) &= |\Psi(t_i)\rangle\langle\Psi(t_i)|\\
&= \rho_{11}(t_i) + \rho_{22}(t_i) + \rho_{21}(t_i) + \rho_{12}(t_i)\,,
\end{aligned}$$

where $\rho_{mn}(t_i) = \left|\psi_m(t_i)\right\rangle\left\langle\psi_n(t_i)\right|$. The terms $\rho_{21}+\rho_{12}$ account for quantum interference. Then, at time t_f, when the partial waves of the charged particles are recombined at the location $\mathbf{q}_f$, it is seen that the interference pattern is described by the diagonal elements of the reduced density matrix $\rho_r(\mathbf{q}_f,\mathbf{q}_f,t_f)$ in Eq. (2.8).

The expression (2.8) of the reduced density matrix at time t_f accounts for the full quantum effects of a nonrelativistic charged particle, but the corresponding path integral can not be carried out without invoking further approximation [4]. Now we consider the charged particle as a well-defined wave packet where its mean trajectory follows the classical path constrained by an appropriate external potential $V(\mathbf{q})$. Since the finite spread of the particle's wave packet, due to uncertainties on both position and momentum, can be legitimately neglected when its de Broglie wavelength, λ_{dB} is much shorter than the characteristic length scale associated with the accuracy of the measurement l. Thus, as long as $l \gg \lambda_{\mathrm{dB}}$, the wave packet can be viewed as sharply peaked in the position and momentum of the charged particle, and thus its intrinsic quantum effects can be safely ignored [4]. As such, the leading effect of the decoherence can be obtained by evaluating the propagating function (2.9) along prescribed classical paths of the chagres. Thereby, the diagonal components of the reduced density matrix $\rho_r(\mathbf{q}_f,\mathbf{q}_f,t_f)$ becomes

$$\rho_r(\mathbf{q}_f,\mathbf{q}_f,t_f) = \left|\psi_1(\mathbf{q}_f,t_f)\right|^2 + \left|\psi_2(\mathbf{q}_f,t_f)\right|^2 + 2\,e^{\mathcal{W}[j_1,j_2]}\,\mathfrak{Re}\left\{e^{i\Phi[j_1,j_2]}\psi_1(\mathbf{q}_f,t_f)\,\psi_2^*(\mathbf{q}_f,t_f)\right\}, \tag{2.20}$$

where the $\mathcal{W}$ and Φ functionals are evaluated along the classical trajectories, C_1 and C_2. The $j_{1,2}$ are the classical current along the respective paths. Thus, we find that the interference contrast is modified by the presence of the functionals $\mathcal{W}[j_1,j_2]$. Explicitly, the decoherence functional $\mathcal{W}$, determined by the Hadamard function of vector potentials, reveals loss of coherence between charged particles, while the phase functional Φ, related to the retarded Green's function, causes an overall phase shift of the interference pattern. Both effects arise from the interaction with the quantized electromagnetic fields.

3 Gauge invaraince of the decoherence functional

The role of the gauge fixing term is most obvious if we express the Green's function in the momentum space,

$$\begin{aligned}\left\langle A^\mu(x)A^\nu(y)\right\rangle &= G^{\mu\nu}(x-y)\\ &= \int\frac{d^4k}{(2\pi)^4}\,G^{\mu\nu}(k)\,e^{-ik\cdot(x-y)}\,.\end{aligned} \tag{3.1}$$

In general covariant gauge, the Green's function is given by

$$G^{\mu\nu}(k) = \frac{1}{k^2}\left(-\eta^{\mu\nu} - (\xi-1)\frac{k^\mu k^\nu}{k^2}\right), \tag{3.2}$$

where k is the four-momentum. For the special choice of the gauge fixing parameter $\xi = 1$, it reduces to the form in Lorenz gauge [8]. We note that if the charged particles are sufficiently localized in space, then the current will be conserved,

$$\partial_\mu j^\mu(x) = 0, \qquad \text{or} \qquad k_\mu j^\mu(k) = 0. \tag{3.3}$$

This is known as the on-shell condition. It is straightforward to see that the decoherence functional $\mathcal{W}$ reduces to the expression derived in Ref. [4], where the momentum-space Green's function is given in Lorenz gauge.

We may further reduce the result to the one derived in Ref. [5]. Let us expand the vector potentials by the creation and annihilation operators in Lorenz gauge,

$$A^\mu(x) = \int \frac{d^3\mathbf{k}}{(2\pi)^{\frac{3}{2}}} \frac{1}{\sqrt{2\omega}} \sum_{\lambda=0}^{3} \varepsilon_\lambda^\mu(\mathbf{k}) a_\lambda(\mathbf{k})\, e^{i\mathbf{k}\cdot\mathbf{x}-i\omega t} + \text{H.C.} \tag{3.4}$$

with $\omega = |\mathbf{k}|$. The polarization vectors ε_λ^μ obey the condition,

$$\sum_{\lambda=0}^{3} \eta_{\lambda\lambda}\, \varepsilon_\lambda^\mu(\mathbf{k}) \varepsilon_\lambda^\nu(\mathbf{k}) = \eta^{\mu\nu}. \tag{3.5}$$

Then, the Hadamard function in Lorenz gauge, denoted by $D_H^{\mu\nu}$, can be explicitly given by

$$\begin{aligned} D_H^{\mu\nu} &= \frac{1}{2}\langle 0 | \left\{A^\mu(x), A^\nu(x')\right\} | 0 \rangle) \\ &= -\pi\eta^{\mu\nu} \int \frac{d^4k}{(2\pi)^4}\, e^{-ik\cdot(x-x')}\, \delta(k^2). \end{aligned} \tag{3.6}$$

However, we observe that, from Eq. (3.5),

$$\eta^{\mu\nu} = \sum_{\lambda=1}^{2} \varepsilon_\lambda^\mu(\mathbf{k}) \varepsilon_\lambda^\nu(\mathbf{k}) + \frac{k^\mu k^\nu - (k\cdot n)[k^\mu n^\nu + n^\mu k^\nu]}{(k\cdot n)^2 - k^2} + \frac{k^2 n^\mu n^\nu}{(k\cdot n)^2 - k^2},$$

where n is some time-like vector. The polarization vectors in Lorenz gauge include the transverse components of the electromagnetic fields in the first term, as well as the longitudinal and the scalar components in the last two terms [9]. It is immediately seen that the second term, proportional to k^μ, will not contribute to the decoherence functional due to charge conservation. In addition, the third term also vanishes since

$$-\pi \int \frac{d^4k}{(2\pi)^4}\, e^{-ik\cdot(x-x')}\, \delta(k^2) \frac{k^2 n^\mu n^\nu}{(k\cdot n)^2 - k^2} = 0,$$

because for a regular function $f(k)$, it is true that

$$\int dk\, f(k)\, k^2 \delta(k^2) = 0\,.$$

Therefore, we explicitly reduce the decoherence functional (2.16) into the form obtained in Coulomb gauge, where only the transverse photons, as the physical degrees of freedom of the electromagnetic fields, contribute.

To further illustrate gauge invariance of the decoherence functional, we plug the explicit expression of the currents (2.2) into the decoherence functional (2.16), and carry out the integrals. Since the two worldlines $\mathcal{C}_1$ and $\mathcal{C}_2$ form a closed loop $\mathcal{C} = \mathcal{C}_1 - \mathcal{C}_2$ by moving along $\mathcal{C}_1$ in the forward direction and $\mathcal{C}_2$ in the backward direction instead, the decoherence functional $\mathcal{W}$ can be re-written as

$$\mathcal{W} = -\frac{1}{2}\, e^2 \oint_{\mathcal{C}} dq_\mu \oint_{\mathcal{C}} dq'_\nu\, D_H^{\mu\nu}(q,q')\,. \tag{3.7}$$

With the help of the Stokes theorem, we may write the loop integral into the form of the surface integral,

$$\mathcal{W} = -\frac{1}{8}\, e^2 \int da_{\mu\nu} \int da'_{\rho\sigma}\, D_H^{\mu\nu;\,\rho\sigma}(q,q')\,, \tag{3.8}$$

where $da_{\mu\nu}$ is the area element of the time-like two-surface enclosed by $\mathcal{C}$, and

$$D_H^{\mu\nu;\,\rho\sigma}(q,q') = \frac{1}{2}\big\langle\big\{F^{\mu\nu}(q), F^{\rho\sigma}(q')\big\}\big\rangle\,. \tag{3.9}$$

Thus, the decoherence functional is expressed in a gauge-invariant way.

4 Evaluation of the $\mathcal{W}$ functional

As a simple example, let us compute the decoherence functional $\mathcal{W}$ when the electromagnetic fields are initially in the vacuum state [4, 10]. The motion of the charged particles can be dictated by an external potential along the prescribed paths. The motion of the charged particle along the x direction is assumed to be constant, while the motion in the z direction varies with time. Thus, their respective worldlines are given by $\mathcal{C}_{1,2} = (t, v_x t, 0, \pm\zeta(t))$. Since the decoherence functional $\mathcal{W}$ in Eq. (3.7) reveals manifest Lorentz invariance, it proves more convenient to boost to a frame comoving with the velocity $u = (1, v_x, 0, 0)$ at $x = v_x t$ and $y = z = 0$, in which the charged particles are seen to only have sideway motion in the z direction. Then, the $\mathcal{W}$ functional (3.7) depends only on the z–z component of the vector-potential Hadamard function. With the help of the Hadamard function in Lorenz gauge (3.6), the straightforward algebraic manipulation shows

$$\mathcal{W} = -2e^2 \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{1}{2\omega} \left[1 - \frac{k_z^2}{\omega^2}\right] \left| \int dt\, \dot{\zeta} \cos(k_z \zeta)\, e^{i\omega t} \right|^2 , \tag{4.1}$$

where $\dot{\zeta} = d\zeta/dt$. We then end up with the same expression of the decoherence functional as in Ref. [5]. We further simplify the calculation by applying the dipole approximation, $\cos(k_z\zeta) \simeq 1$, consistent with the non-relativistic limit. Using the path function of the form

$$\zeta(t) = R\, e^{-\frac{t^2}{T^2}} , \tag{4.2}$$

where $2R$ is the effective path separation and $2T$ is the effective flight time, we have the decoherence functional given by

$$\begin{aligned} \mathcal{W} &\simeq -2e^2 \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{1}{2\omega} \left[1 - \frac{k_z^2}{\omega^2}\right] \left| \int dt\, \dot{\zeta}\, e^{i\omega t} \right|^2 \\ &= -\frac{2e^2}{3\pi} \frac{R^2}{T^2} \left(\frac{1}{c^2}\right) , \end{aligned} \tag{4.3}$$

which is finite without the ultraviolet divergence. The absence of the potential ultraviolet divergence can be seen from the corresponding Fourier transform of the path function (4.2) where the contribution from the high frequency modes with $\omega \gtrsim O(1/T)$ is exponentially suppressed. The result free of ultraviolet divergence is quite general for the smooth path function with the finite flight time.

In the nonrelativistic limit, since the transverse component of particle's velocity $v_z \sim R/T$ is about $10^{-2}c$ in a typical interference experiment, the decoherence functional $\mathcal{W}$, proportional to v_z^2, will be of the order of 10^{-5} to 10^{-6}. Therefore, the loss of the interference contrast due to vacuum fluctuations of quantized electromagnetic fields may be still far from being measurable.

5 Concluding remarks

The aim of this paper is to present a field-theoretic approach to investigate decoherence between charged particles due to the quantized electromagnetic fields by the method of Feynman-Vernon influence functional. We have shown that the influences of the quantized electromagnetic fields on particle's interference are manifested in both modification of the fringe contrast and shift of the interference pattern. Both effects arise from the interaction of the quantized electromagnetic fields with the charged particle. We also have explicitly shown that the incorporation of the gauge-fixing term in general covariant gauge, which is used to remove some of gauge redundancy, has no contribution to the decoherence functional owing to charge conservation. Furthermore, the equivalence between the expressions of the decoherence functional in general covariant gauge and the Coulomb gauge is demonstrated. Finally, we rewrite the decoherence functional into the gauge-invariant form in terms of the field-strength tensor.

The extension of the above study involving thermal fluctuations within the same formalism is in progress.

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在一般協變規範下與量子化電磁擾動藕合的帶電粒子之非平衡有效作用量

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摘　　要

與量子化電磁場作用的帶電粒子的非平衡有效作用量，可用閉時路徑公式，在一般協變規範下，藉由 積分掉場變數而求得。由此可得帶電粒子的化約密度矩陣，並藉著在電磁量子化擾亂之影響下的干涉實驗，可根據去同調函數 W 的觀點探討 帶電粒子的 量子去同調現象。在本論文中，亦討論到去同調函數的規範不變性。